School of Mathematics, Thapar Institute of Engineering & Technology, Patiala
Mid-Term Examination, March 2023

B.E. IV Semester
Time Limit: 02 Hours
Instructor(s) : Dr. Arvind K. Lal, Dr. Paramjeet Singh, Dr. Sanjeev Kumar

UMA011 : Numerical Analysis
Maximum Marks: 25

Instructions: You are expected to answer all question. Arrange your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Scientific Calculator is permitted.

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1. (a) By calculating the condition number of all functions involved in expression $f(x) = \ln \sqrt{x}$, show that this expression is stable for $x = 123$. [3 marks]

   (b) Show that the bisection method converges linearly to the simple root of the equation $f(x) = 0$. [2 marks]

2. (a) The equation $e^x - 4x^2 = 0$ has a root in $[4, 5]$. Show that we cannot find that root using fixed-point method by taking $g(x) = \frac{1}{2}e^{x/2}$. Can you find another $g(x)$ which will locate that root? If yes, then find three iterations with initial guess $x_0 = 4.2$. [4 marks]

   (b) Apply Newton's method to find the positive root of the equation $\cos x - x^2 = 0$ correct to three decimals by taking initial guess $x_0 = 0.8$. [4 marks]

3. (a) Solve the following system using the Gauss elimination method:

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 2x_1 + 3x_2 + 4x_3 &= 3 \\ 4x_1 + 9x_2 + 16x_3 &= 11. \end{aligned}$$

[4 marks]

   (b) Find the condition number of the following matrix in max-norm (infinity-norm):

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 3 & -1 & 4 \end{bmatrix}.$$

[3 marks]

4. Consider the following system of linear equations:

$$\begin{aligned} 2x_1 - x_2 + x_3 &= -2 \\ x_1 + x_2 + x_3 &= 2 \\ -x_1 - x_2 - 2x_3 &= -3. \end{aligned}$$

   (a) Is the coefficient matrix strictly diagonally dominant? Justify your answer. [2 marks]

   (b) By calculating the value of iteration matrix T_g of the Gauss-Seidel method, show that the method converges. [3 marks]